

Problem 1.47

[Difficulty: 3]

1.47 The estimated dimensions of a soda can are $D = 66.0 \pm 0.5$ mm and $H = 110 \pm 0.5$ mm. Measure the mass of a full can and an empty can using a kitchen scale or postal scale. Estimate the volume of soda contained in the can. From your measurements estimate the depth to which the can is filled and the uncertainty in the estimate. Assume the value of SG = 1.055, as supplied by the bottler.

Given: Soda can with estimated dimensions $D = 66.0 \pm 0.5$ mm, $H = 110 \pm 0.5$ mm. Soda has SG = 1.055

Find: Volume of soda in the can (based on measured mass of full and empty can); Estimate average depth to which the can is filled and the uncertainty in the estimate.

Solution: Measurements on a can of coke give

$$m_f = 386.5 \pm 0.50 \text{ g}, \quad m_e = 17.5 \pm 0.50 \text{ g} \therefore m = m_f - m_e = 369 \pm u_m \text{ g}$$

$$u_m = \left[\left(\frac{m_f}{m} \frac{\partial m}{\partial m_f} u_{m_f} \right)^2 + \left(\frac{m_e}{m} \frac{\partial m}{\partial m_e} u_{m_e} \right)^2 \right]^{\frac{1}{2}}$$

$$u_{m_f} = \pm \frac{0.5 \text{ g}}{386.5 \text{ g}} = \pm 0.00129, \quad u_{m_e} = \pm \frac{0.50}{17.5} = 0.0286$$

$$u_m = \pm \left[\left(\frac{386.5}{369} \times 1 \times 0.00129 \right)^2 + \left(\frac{17.5}{369} \times 1 \times 0.0286 \right)^2 \right]^{\frac{1}{2}} = \pm 0.0019$$

Density is mass per unit volume and SG = $\rho/\rho_{\text{H}_2\text{O}}$ so

$$V = \frac{m}{\rho} = \frac{m}{\rho_{\text{H}_2\text{O}} \text{ SG}} = 369 \text{ g} \times \frac{\text{m}^3}{1000 \text{ kg}} \times \frac{1}{1.055} \times \frac{\text{kg}}{1000 \text{ g}} = 350 \times 10^{-6} \text{ m}^3$$

The reference value $\rho_{\text{H}_2\text{O}}$ is assumed to be precise. Since SG is specified to three places beyond the decimal point, assume $u_{\text{SG}} = \pm 0.001$. Then

$$u_v = \left[\left(\frac{m}{v} \frac{\partial v}{\partial m} u_m \right)^2 + \left(\frac{\text{SG}}{v} \frac{\partial v}{\partial \text{SG}} u_{\text{SG}} \right)^2 \right]^{\frac{1}{2}}$$

$$u_v = \pm \left[(1 \times 0.0019)^2 + (-1 \times 0.001)^2 \right]^{\frac{1}{2}} = \pm 0.0021 = \pm 0.21\%$$

$$V = \frac{\pi D^2}{4} L \quad \text{or} \quad L = \frac{4V}{\pi D^2} = \frac{4}{\pi} \times \frac{350 \times 10^{-6} \text{ m}^3}{0.066^2 \text{ m}^2} \times \frac{10^3 \text{ mm}}{\text{m}} = 102 \text{ mm}$$

$$u_L = \left[\left(\frac{\forall}{L} \frac{\partial L}{\partial \forall} u_{\forall} \right)^2 + \left(\frac{D}{L} \frac{\partial L}{\partial D} u_D \right)^2 \right]^{\frac{1}{2}}$$

$$\frac{\forall}{L} \frac{\partial L}{\partial \forall} = \frac{4}{\pi D^2} \frac{\pi D^2}{4} = 1$$

$$\frac{D}{L} \frac{\partial L}{\partial D} = \frac{D}{L} \cdot -2 \frac{4\forall}{\pi D^3} = -2 \frac{4\forall}{\pi D^2 L} = -2; \quad u_D = \pm \frac{0.5}{66} = \pm 0.0076$$

$$u_L = \pm \left[(1 \times 0.0021)^2 + (-2 \times 0.0076)^2 \right]^{\frac{1}{2}} = \pm 0.0153 = \pm 1.53\%$$

Notes:

1. Printing on the can states the content as 355 ml. This suggests that the implied accuracy of the SG value may be over stated.
2. Results suggest that over seven percent of the can height is void of soda.